

Assignment 3

This homework is due Friday Feb 13.

There are total 50 points in this assignment. 45 points is considered 100%. If you go over 45 points, you will get over 100% for this homework (but not over 115%) and it will count towards your course grade.

Collaboration is welcome. If you do collaborate, make sure to write/type your own paper *and give credit to your collaborators in your pledge*. Your solutions should exhibit your work and contain full proofs. Bare answers will not earn you much.

This assignment covers Sections 1.6–2.1 of Textbook.

- (1) [5pt] Find a parametrization of a curve that is a straight line (a) from the point $-i$ to the point $1 + i$, (b) from the point 2 to the point $1 + i$.
- (2) [5pt] Sketch the curve $z(t) = t^2 + 2t + i(t + 1)$ (a) for $-1 \leq t \leq 0$, (b) for $1 \leq t \leq 2$. (*Hint*: Use $x = t^2 + 2t$, $y = t + 1$ to eliminate t , therefore getting a condition on x, y .)
- (3) [5pt] Find a parametrization of the curve that is a portion of the circle $|z - 1| = 1$ that joins $1 - i$ to $1 + i$ if (a) the curve is the right semicircle, (b) the curve is the left semicircle.
- (4) [5pt] Find $f(-1 + i)$ for the following functions
 - (a) $f(z) = z + z^{-2} + 5$.
 - (b) $f(z) = \frac{1}{z^2 + 1}$.
 - (c) $f(z) = f(x + iy) = x + y + i(x^3y - y^2)$.
 - (d) $f(z) = z^2 - 4z\bar{z} - 5\operatorname{Re}(z) + \operatorname{Im}(z)$.
 - (e) $f(z) = z^{19} - 4z^8 + 9z^3$ (in this item, use polar coordinates).
- (5) [5pt] Express the following functions in the form $u(x, y) + iv(x, y)$.
 - (a) $f(z) = z^3$.
 - (b) $f(z) = \bar{z}^2 + (1 - 4i)z$.
 Express the following functions in the polar coordinate form $u(r, \theta) + iv(r, \theta)$.
 - (c) $z^5 + \bar{z}^5$,
 - (d) $z^3 + z^{-3}$.
- (6) [5pt] Let $f(z) = f(x + iy) = e^x \cos y + ie^x \sin y$. Find
 - (a) $f(0)$, (b) $f(i\pi)$, (c) $f(i\frac{2\pi}{3})$, (d) $f(2 + i\pi)$, (e) $f(3\pi i)$.
 Is f a one-to-one function? Explain your answer.
- (7) [5pt] For $z \neq 0$, put $f(z) = \ln r + i\theta$, where $r = |z|$, and $\theta = \operatorname{Arg} z$. Find
 - (a) $f(1)$, (b) $f(-2)$, (c) $f(1 + i)$, (d) $f(-\sqrt{3} + i)$.
 Is f a one-to-one function? Explain your answer.
- (8) [15pt] Let $w = f(z) = (3 + 4i)z - 2 + i$.
 - (a) Find the image of the disk $|z - 1| < 1$.
 - (b) Find the image of the line $x = t, y = 1 - 2t$ for $-\infty < t < \infty$.
 - (c) Find the image of the half-plane $\operatorname{Im}(z) > 1$.
 - (d) For parts (a), (b), and (c) sketch the mapping (that is, sketch the region and its image). In each case identify three points of your choice and their corresponding images.